Effect of Periodic Unit Cell Volume on Attenuation Zones of 1D-Meta-material-based Periodic Foundations

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ABSTRACT

The one-dimensional (1D) meta-material-based periodic foundations (PFs) can attenuate seismic waves at the base of the super-structure in both horizontal and vertical directions provided their frequency bandgaps lies within the predominant frequency range of ground motions. The periodic unit cells in 1D PF should be made of a high-density rigid material layer adjacent to a low-shear modulus soft material layer for attaining low starting frequencies (SF) with wider bandgaps (BG). However, such high-density rigid layers can be uneconomical while low-shear modulus soft layer are laterally unstable. The analytical formulations such as the Plane Wave Expansion Method and the Transfer Matrix Method for the calculation of SF and BG consider infinite plan area of the layers, which ignores the effect of lateral instability and volume of the layers. Thus, in this study, a mass-spring chain approach (MSCA) has been adopted to conceptualize the continuous layered PF into a discrete system and investigate the effect of volume and stiffness of material layers. Thereafter, a finite element approach on the PFs was used to (1) examine the effect of steel shims in soft rubber layers and (2) the effect of rigid layers with constant volume varying stiffness on wave propagation through PF. The study concluded that the increase in the shape factor of soft rubber improved the wave propagation through PF resulting in a well-shaped FRF (due to improved lateral stability). The bandgap characteristics (SF and BG) of the PFs depend only on the volume (irrespective of shape or size) of the rigid layer while the soft layer depends both on stiffness and volume.

1 INTRODUCTION

Base isolation is a well-known technique to protect structural and non-structural components of a building from the damaging effects of seismic waves. Generally, base isolation can be accomplished by either modifying the fundamental properties of structures (typically the foundation) such that the seismic waves are less destructive to the improved structure (Kelly 1986; Jangid and Datta 1995; Naeim and Kelly 2000; Jangid 2015) or by attenuating the seismic waves before they reach the super-structures (Mu et al. 2020). The former is a well-established technique that utilizes rubber bearings and sliding bearings at the base of the buildings to lower their fundamental frequency. The bearings are often limited to provide horizontal isolation (Kumar and Kumar 2021).

The one-dimensional (1D) periodic foundations (PFs) that are based on the Bragg scattering principle are the most practical form of seismic meta-materials that can be used on construction sites. The 1D PFs consist of periodically repeating unit cells, made of layers of softer and rigid materials arranged in a specific manner as shown in Figure 1. The material properties and the thickness of the layers in unit cells control the range of frequency that can be filtered out. Researchers (Jain et al., 2020; Witarto et al., 2016) conducted parametric studies on the material and geometric properties of the layers and found that to filter out waves with low-frequency range of 0 Hz- 50 Hz (predominant frequency range of earthquakes) alternate high-density rigid layer is required adjacent to low shear modulus soft layer. The contrast in properties of adjoining material layers is necessary to create conditions of negative refraction for the incoming waves and consecutively leading to destructive interference of waves.

Experimental and numerical investigations confirmed the wave filtering ability of the 1D PFs (Xiang et al. 2012; Huang et al. 2021; Zhao et al. 2021; Wang et al. 2022), however no attention was paid to the lateral stability of extremely soft layer in combination with high density rigid layer. Steel and rubber are commonly used construction materials that are adopted for the rigid and softer layers in PFs, respectively. The bulging effect of thick, soft rubber layers can be stabilised by increasing its shape factor (Buckle et al. 2002). However, the requirement of thick steel material for rigid layers is not economical. The analytical formulations such as the transfer matrix method (TMM) and the plane wave expansion method (PWE) assume infinite plan dimensions for the calculation of SF and BG (Hussein et al. 2014; Banerjee et al. 2019).

Thus, to study the effect of the volume and stiffness of layers on the bandgap characteristics of finite sized periodic unit cells, a mass-spring chain approach (MSCA) was adopted. The validatory results with TMM showed that higher order discretization was required contrary to a single lumped-mass-spring for each layer of PF. Once the validation was completed, the volume and stiffness effect of the layers was evaluated on bandgap characteristics of PFs. The volume (or mass) of rigid layers was found to be more sensitive to SF and BG rather than stiffness (assuming that the stiffness of the rigid layer is much greater than the softer layer). On the other hand, the SF and BG of the PF is dependent on both volume and stiffness of the softer layer in the unit cells of PF. Finite element analysis (FEA) has also been carried out to study the effect of steel shims in soft rubber layers and the effect of using rigid layers of varying cross sections with different shapes and sizes but constant volume on the wave propagation through PF. The wave propagation through PF was found to be affected due to partial bonding of softer layer with rigid layer at the cross-section level while the steel shims improved the wave propagation.

2 ONE-DIMENSIONAL MASS-SPRING CHAIN APPROACH

2.1 Theory of MSCA

The bandgap characteristics of continuous PFs can be conceptualised into a mass-spring chain system with equivalent mass (m_j) and stiffness (k_j) of layers as shown in Figure 1 (Jensen 2003; Wen et al. 2008; Cheng and Shi 2014). The governing equation for the j^{th} mass of the unit cell is shown in equation (1). The BG frequencies of the infinitely repeated mass-spring unit cell with N-degree freedom can be evaluated (using dispersion curves) after solving the eigenvalue problem of the characteristic equations (equation (2)) within the irreducible Brillouin zone (Jensen 2003).

$$m_j \ddot{u}_j = k_j (u_{j+1} - u_j) - k_{j-1}(u_j - u_{j-1}) \tag{1}$$

$$(\omega_j^2 - \omega^2)A_j = c_j^2 e^{ik} A_{j+1} + (\omega_j^2 - c_j^2) e^{-ik} A_{j-1} \quad (2)$$

Where, ω is radial frequency of a wave, A_j is amplitude of a wave, $c_j = \frac{k_j}{m_j}$, $\omega_j^2 = \frac{k_{j+}k_{j-1}}{m_j}$

The P waves and the S waves (body waves) are responsible for vertical and horizontal ground motions during an earthquake. A PF with bi-layered unit cell made of steel and rubber has been considered in the present study to investigate the effect of volume and stiffness of rigid and soft layers respectively under P waves. The material properties adopted for the steel and rubber are shown in Table 1. A finite plan dimension of 0.4 m × 0.4 m and a thickness of 0.2 m has been assumed for each layer that is feasible for construction of foundations. The SF and BG using the MSCA have been evaluated and plotted as shown in Figure 2 with increasing order of discretization (denoted by degrees of freedom (DOF)) for each layer in the unit cell. It has been observed that the number of DOF does not affect the SF of the bi-layered PF. However, the BG first increased with the number of DOF of the individual layers in the unit cell (up to 10 DOF) and thereafter remained constant. Hence, each layer has been discretized using 10 DOF for further analysis.

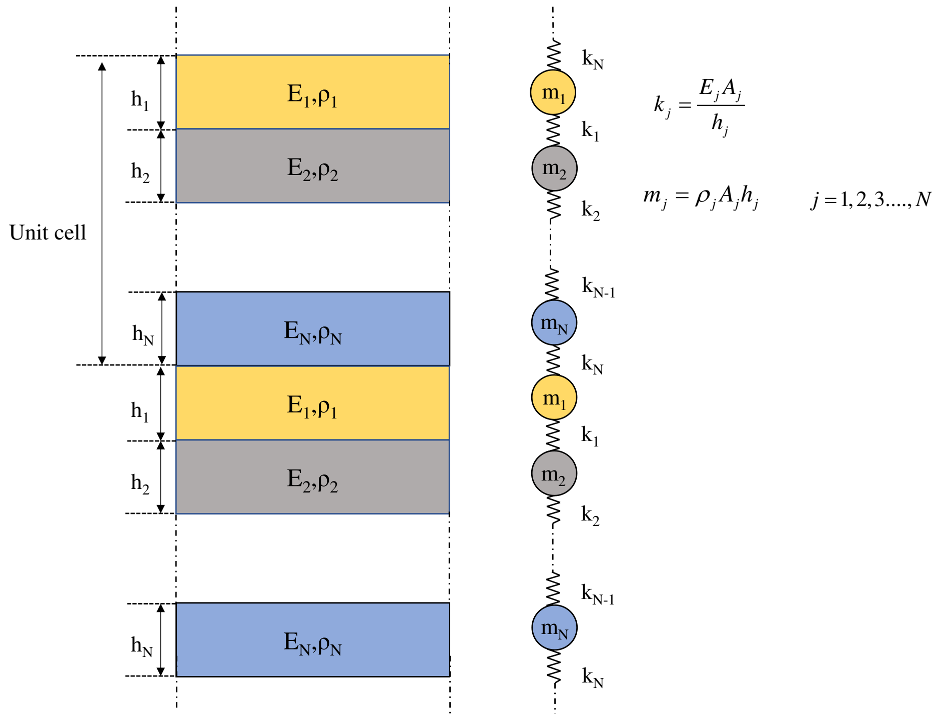

Figure 1 Conceptualisation of bi-layered continuous PF into the MSCA

Table 1 Material Properties of bi-layered continuous PF

Material	G (MPa)	ρ (kg/m³)	ν	λ (MPa)	M (MPa)
Steel	**80769**	**7850**	**0.3**	**121153**	**282691**
Rubber	**0.05**	**1200**	**0.463**	**0.678**	**0.778**

Where, G = Shear Modulus, ρ = Density, ν = Poisson's ratio, λ = Lame's 1st parameter, M = Constrained compression modulus

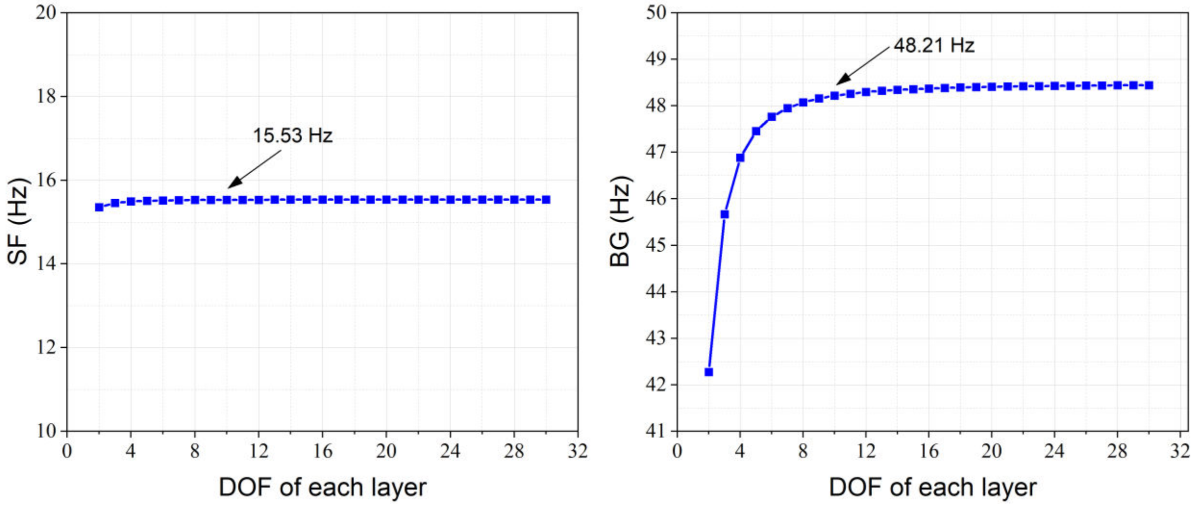

Figure 2 Effect of finer discretization (higher DOF) of individual layers of PF on SF and BG

2.2 Effect of Volume and Stiffness of Layers in PF

The volume and stiffness effect of rigid and softer layers in unit cells of PF is investigated in this section. A parametric study on PF with rubber-steel unit cell of individual layer size 0.4 m × 0.4m × 0.2m (as stated in Section 2.1) has been considered. The volumetric effect of finite-sized layers of the unit cell has been studied by varying the volume of the two material layers ($V=A\times h$) such that the stiffness ($K=MA/h$) remains constant to its initial value for various heights of layers. Figure 3 shows the variation of SF and BG with increasing heights (i.e., increasing volume) of the rigid and soft layers. It can be observed from Figure 3 that the SF decreases but the BG increases with an increase in the volume of the rigid layer. However, both the SF and BG has been observed to reduce with an increase in the volume of the softer layer. The initial value of SF=15.53 Hz and BG= 48.21 Hz for layer height of 0.2 m can be seen in Figure 3. The volume effect of the layers has been found to be consistent with the density effect of the layers in unit cells of PF (Witarto et al. 2016).

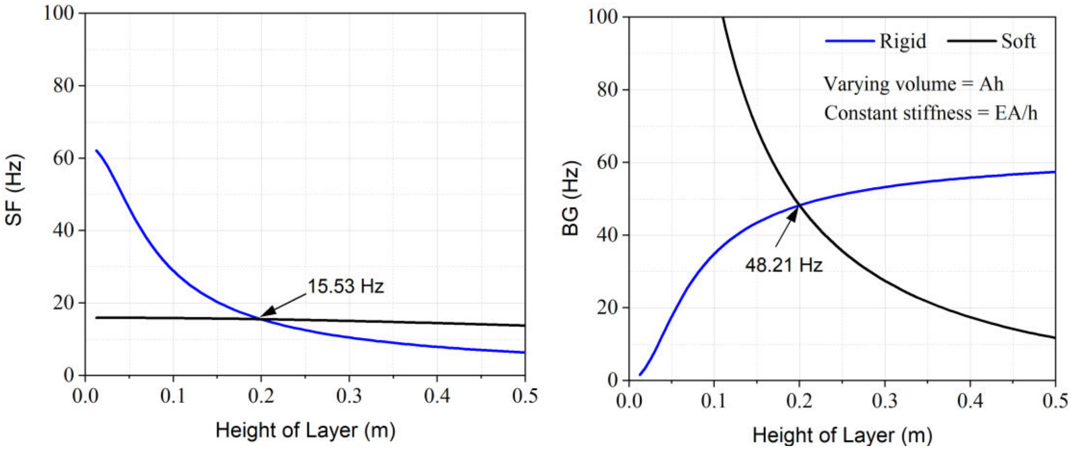

Figure 3 Effect of Volume of Layers on Bandgap Characteristics

Similar to the volumetric effect, the effect of varying stiffness has been studied by altering ($K=MA/h$) such that the volume of the layers ($V=A\times h$) remains constant to its initial value for various heights of layers. Figure 4 shows the variation of SF and BG with increasing heights (i.e., decreasing stiffness) of the rigid and soft layers. The stiffness of the rigid layer has no impact on the SF and the BG. However, both the SF and BG have been observed to increase with an increase in the stiffness of the softer layer. The effect of stiffness on the SF and the BG is in line with the effect of elastic modulus of layers in unit cells of PF (Witarto et al. 2016; Jain et al. 2020).

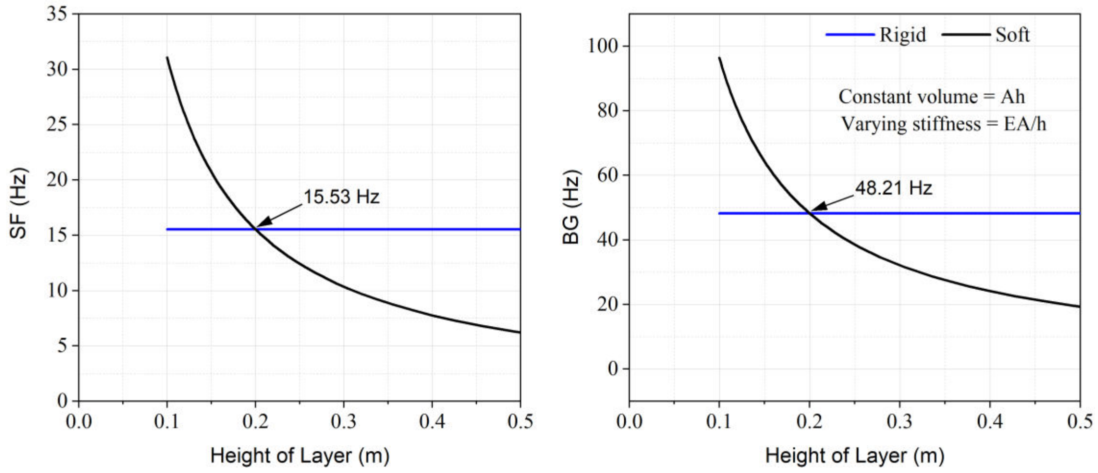

Figure 4 Effect of Stiffness of Layers on Bandgap Characteristics

3 FINITE ELEMENT ANALYSIS

The same PF as discussed in Section 2 is simulated in finite element software ABAQUS with finite plan area having two rubber-steel unit cells. C3D8 solid elements with linear elastic material properties have been assigned to the developed models. The bandgap characteristics of the finite sized PFs have been evaluated in terms of the frequency response function (FRF) which is given by the logarithmic base 10 value of the ratio of the response at the top of PF to the input excitation at the base of PF. The four-layered finite sized PF with two-unit cells has been observed to have jagged FRF due to the lateral bulging of rubber. Thus, the effect of increasing the shape factor of the rubber layers in PF has been investigated initially in Section 3.1 in order to enhance the bandgap characteristics and improve the lateral stability of rubber. Thereafter, the effect of constant volume and varying stiffness of rigid steel layers has been investigated in Section 3.2 by considering steel layers of various cross sections and shapes.

3.1 Effect of Steel Shims in Soft Rubber Layer

The requirement of soft, thick rubber layers in PFs (for lower SF) creates lateral instability due to bulging. The shape factor (S) has been found to play a major role in enhancing the properties of rubber. Thus, steel shims have been placed in the rubber layers of PF to investigate the impact of the shape factor on enhancing the bandgap characteristics through FRF. Steel shims ranging from number 0 to 5 were positioned at regular intervals, raising the shape factor of rubber from 0.5 to 3. The FRFs for various shape factors are plotted in Figure 5. It is clear from Figure 5 that the jagged FRF corresponding to lower shape factor (represented by the dotted lines) evolves into well-shaped FRFs (represented by the solid lines) showing the peak resonance frequencies and the BG region which concurs with the analytical FRF obtained using TMM (for S=0.5 and infinite plan dimensions). Thus, the shape factor does contribute significantly to increasing the stability of rubber by eliminating the lateral vibration modes from the FRF.

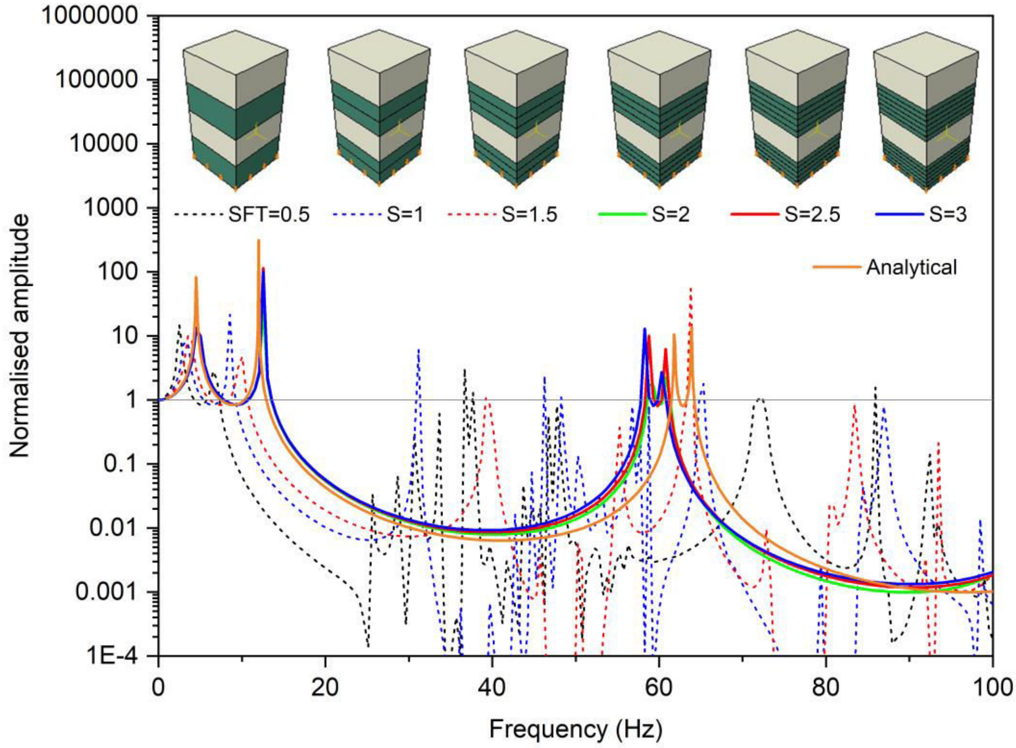

Figure 5 Effect of shape factor on FRF of PF

3.2 Effect of Varying Cross-Section of Rigid layer

This exercise gives theoretical insights in effect of different shape configuration of rigid layers with constant volume and varying stiffness in unit cells of PFs. The PF with S=3 has been selected for further analysis and retained as the benchmark (BM) case. In the Trial 1 of the exercise, various percentages of hollowness (6.3%, 25%, and 56%) have been introduced in the steel layers having constant material volume. The FRFs for the individual cases in Trial 1 have been plotted as shown in Figure 6 to show the constant outcome of bandgap characteristics. In other words, using hollow steel sections rather than solid steel sections with the same plan area and layer height can alter the wave filtering ability of PF. Also, the wave propagation in the PF is found to be affected by the partial bonding of the soft layer with the rigid layer (due to hollowness in rigid layer), which resulted in the formation of kinks in the FRF (Figure 6).

Further, four cases have been considered in the second trial (Trial 2) in which the cross sectional area of steel layer with constant volume have been either stretched or compressed (instead of making the cross sections hollow). The cross sectional area of the PF in the four cases considered in the present study have been taken as 25%, 56%, 156% and 225% of the BM. The FRF of all the investigated cases plotted in Figure 7 shows that the bandgap characteristics again remained constant. In cases where the plan dimensions of the rigid layer is larger than the softer layer (complete bonding of soft layer with rigid layer) (BM, 1.56BM, 2.25 BM), the FRFs precisely overlap each other. However, additional kinks were still observed in the FRF of the cases where there is partial bonding of the softer layer with the rigid layer (0.25BM, 0.56BM), thus affecting the wave propagation.

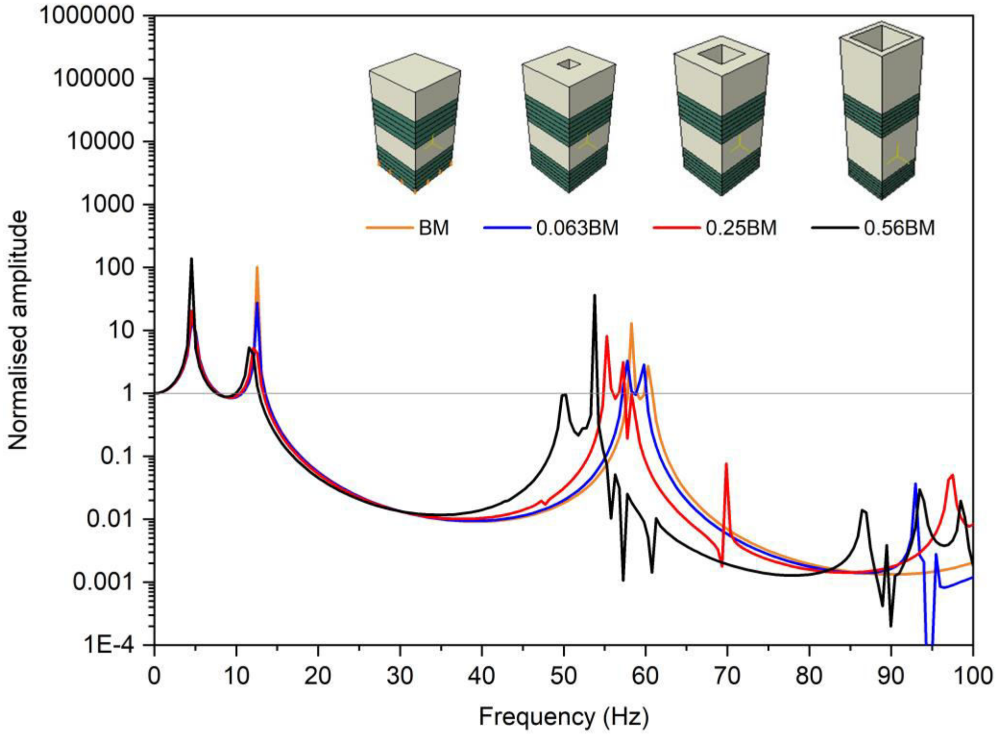

Figure 6 Effect of Hollowness in Rigid Layers of Constant Volume

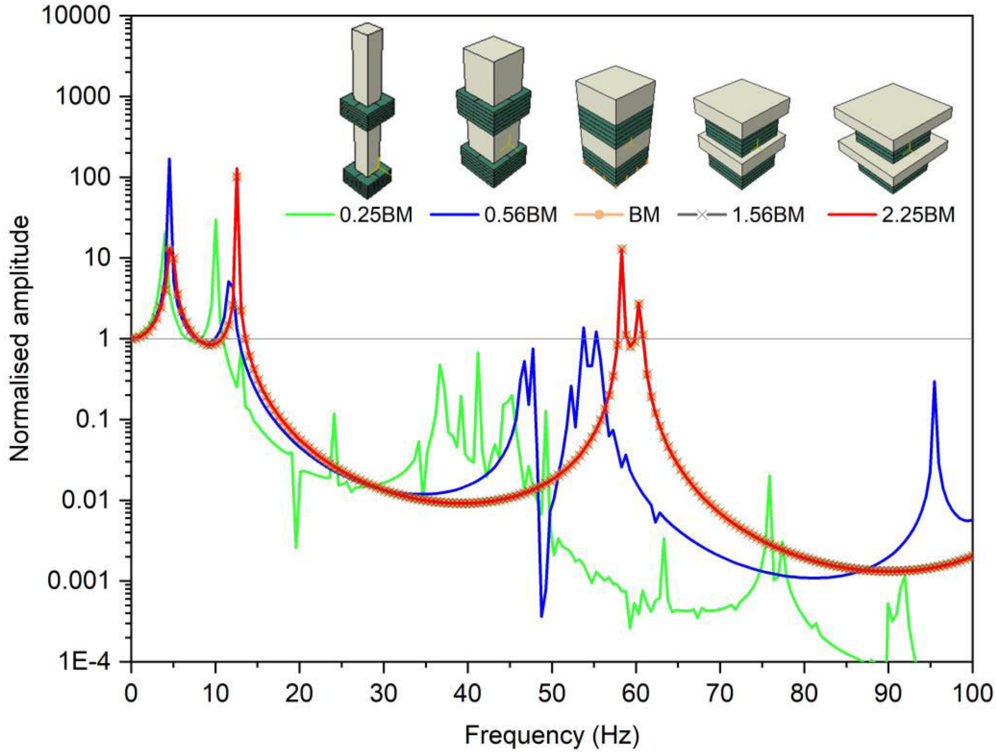

Figure 7 Effect of Stretching or Compressing Rigid Layer Cross Section of Constant Volume

4 SUMMARY AND CONCLUSION

The attenuation of seismic waves at the base of the super structure using PF is an innovative technique of protecting structures and components. The material and geometric properties of PFs required for attenuation of waves with predominant frequency of earthquake energy are absurd from stability and economic point of view. The stability of PFs can however be achieved by reinforcing the soft rubber layers with steel shims. A finite element analysis demonstrated the proficiency of steel shims in increasing the lateral stability of rubber layers in PFs (enhancing the propagation of waves in finite plan sized PF). The main issue with the requirement of thick rigid layers in PF still persists. This study therefore investigated the volume and stiffness effect of layers to check the possibility of using different sizes and shapes of rigid layers while still retaining similar bandgap characteristics. At first, a MSCA was adopted for estimating SF and BG by discretizing the continuous unit cells of PF into equivalent mass spring systems. It has been observed that the layers of adopted PF required at least 10 DOF discretisation for the convergence of SF and BG. The effect of volume and stiffness of layers in PFs were found to be in line with the effect of density and elastic modulus in a infinitely spanned PF. After that, finite element analysis was carried out to demonstrate the effect of various shape configuration of rigid layers with constant volume and varying stiffness. It was found that the wave propagation characteristics were stable unless there exist partial bonding of soft layer with rigid layer. Major conclusions made from the present study are as follows:

1. Higher order discretization is required for convergences of SF and BG in MSCA.
2. The volume and stiffness effect in finite plan sized PF is equivalent to density and elastic modulus effect in infinitely spanned PF.
3. The increase in shape factor of rubber layer improves the lateral instability of PFs.
4. The BG characteristics for a PF remains the same unless the volume of rigid layer is altered irrespective of any shape of cross section.
5. The wave propagation characteristics are affected by the partial bonding of soft layer with rigid layers of PF.

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